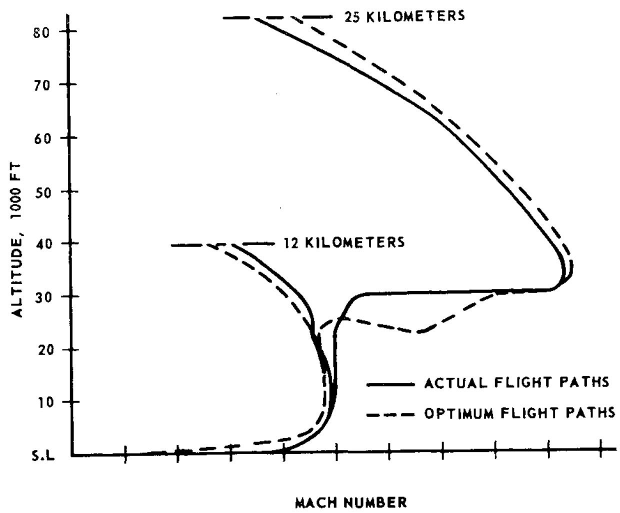

Fig. 4 Comparison of actual and calculated optimum flight paths for two F-4B time-to-climb record flights.

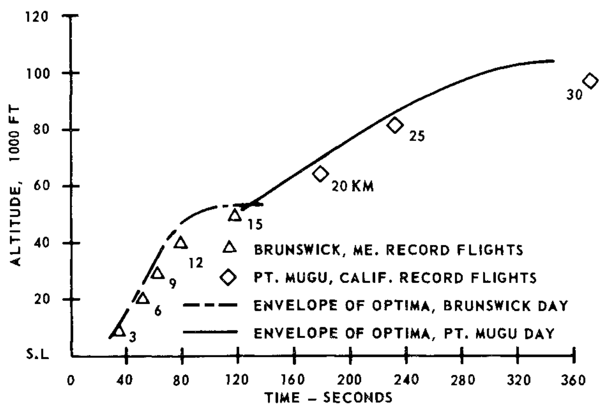

Fig. 5 Actual and calculated time to climb.

time to intercept moving targets or programing optimum turns to acquire targets during which target aspect and missile capability must be taken into account as well as airplane performance. That these calculated optimum paths are substantiated in practice is shown in Fig. 3, which compares an F-4B minimum-time path, flown by Marine Col. Yunck, with the calculated optimum. Both the predicted path and the one actually flown were about 23% better than the corresponding profile prescribed by the flight manual.

Another situation in which optimum operational procedures are required is in planning for record flights such as the F-4B time-to-climb records in the Spring of 1962. The flights to the lower altitudes (3–15 km) were made at Brunswick, Maine, and to the higher altitudes (20–30 km) at Point Mugu, Calif., in order to take advantage of atmospheric conditions existing in each location.

At the time that the records were achieved, the M.A.C. flight path optimization program was not in existence, and flight programing guidance was obtained by means of a simpler performance program on a "cut-and-try" basis to search out the optimum path. This process consumed considerable computing time. Recently, the optimization program now available was used to compute minimum-time paths to various altitudes. Several of these paths shown in Fig. 4 are compared to the actual record flights to 12 and 25 km and show that the flights were flown in a near-optimum manner. Envelopes of the computed minimum times to altitude for both Brunswick and Point Mugu days are given in Fig. 5. Work is continuing at M.A.C. to improve this program by shortening the required computing time and expanding its capability and versatility.

An Evaluation of the Far Field Overpressure Integral

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Nomenclature

$A(t)$ = cross-sectional area of vehicle, ft^2
$B(t)$ = equivalent cross-sectional area due to lift, ft^2
c = coefficient of $(t_N - t_{N-1})$
$F(\tau)$ = effective area-distribution function given by Eq. (2)
K = reflection factor
l = length of vehicle, ft
M = Mach number
n = number points used to subdivide the axis of the vehicle for numerical integration, see Eq. (3)
p = reference pressure, psf
τ_0 = value of τ giving the maximum positive value of $\int_0^\tau F(\tau)d\tau$
t = distance along longitudinal axis from nose of vehicle, ft
y = vertical distance of vehicle from point where overpressure is measured, ft
β = $(M^2 - 1)^{1/2}$
γ = ratio of specific heats
Δp = sonic boom overpressure, psf
τ = dummy variable of integration or upper limit of integration
$(\ )'$ = first derivative
$(\ )''$ = second derivative

Introduction

THE maximum sonic boom overpressure in the far field for a smooth, slender configuration in supersonic flight is given by (these equations are given in their dimensionless form in Ref. 1)

$$\frac{(\Delta p/p)_{\max}(y/l)^{3/4}}{\beta^{1/4}K} = \frac{2^{1/4}\gamma}{l^{3/4}(\gamma+1)^{1/2}}\left(\int_0^{\tau_0} F(\tau)d\tau\right)^{1/2} \quad (1)$$

where

$$F(\tau) = \frac{1}{2\pi}\int_0^\tau A''(t)(\tau - t)^{-1/2}dt + \frac{\beta}{4\pi}\int_0^\tau B''(t)(\tau - t)^{-1/2}dt \quad (2)$$

For most practical configurations the cross-sectional area distribution $A(t)$ and the cross-sectional area equivalent to lift $B(t)$ are defined only at a finite number of points along the axis of the vehicle. This means that, to evaluate the two integrals in Eq. (2), it is necessary to use a curve-fitting technique to obtain an equation for $A(t)$ and $B(t)$. The method of Ref. 1 combines $A(t)$ and $B(t)$ into one equation by fitting the curve $A(t) + B(t)$ with a series of parabolic arcs. This fitted curve is then differentiated twice, and Eq. (2) is evaluated. The purpose of the present paper is to derive equations that avoid the necessity of calculating derivatives of the approximating functions to $A(t)$ and $B(t)$. An equation that does not involve derivatives has been derived in Ref. 2, using a different mathematical approach from that used herein.

Table 1 Comparison of the maximum sonic boom overpressure parameter calculated by Eq. (3) with the exact solution

Configuration 1		Configuration 2	
Exact	Eq. (3)	Exact	Eq. (3)
0.0371	0.0371	0.0687	0.0686

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Derivation

The two integrals in Eq. (2) are the convolution of the functions $A''(\tau)$ and $\tau^{-1/2}$ and $B''(\tau)$ and $\tau^{-1/2}$, respectively[3]; that is,

$$F(\tau) = (1/2\pi)A''(\tau)*\tau^{-1/2} + (\beta/4\pi)B''(\tau)*\tau^{-1/2}$$

Taking the Laplace transform of both sides of the preceding equation gives

$$f(s) = \frac{1}{2\pi}\left[s^2a(s) - sA(0) - A'(0)\right]\frac{\Gamma(\frac{1}{2})}{s^{1/2}} + \frac{\beta}{4\pi}\left[s^2b(s) - sB(0) - B'(0)\right]\frac{\Gamma(\frac{1}{2})}{s^{1/2}}$$

Dividing by s and using the boundary conditions $A'(0) = A(0) = B'(0) = B(0) = 0$ we have

$$\frac{1}{s}f(s) = \frac{1}{2\pi}\,sa(s)\,\frac{\Gamma(\frac{1}{2})}{s^{1/2}} + \frac{\beta}{4\pi}\,sb(s)\,\frac{\Gamma(\frac{1}{2})}{s^{1/2}}$$

Taking the inverse Laplace transform of both sides of this equation gives

$$\int_0^\tau F(\tau)d\tau = \frac{1}{2\pi}\int_0^\tau A'(t)(\tau - t)^{-1/2}dt + \frac{\beta}{4\pi}\int_0^\tau B'(t)(\tau - t)^{-1/2}dt$$

We can write the two integrals on the right side of the preceding equation in the form of Steiltjes integrals

$$\int_0^\tau F(\tau)d\tau = \frac{1}{2\pi}\int_0^\tau(\tau - t)^{-1/2}d[A(t)] + \frac{\beta}{4\pi}\int_0^\tau(\tau - t)^{-1/2}d[B(t)]$$

For automatic computation this equation would be written in the form

$$\int_0^\tau F(\tau)d\tau = \frac{1}{2\pi}\sum_{i=0}^{N}(\tau - t_i)^{-1/2}[A(t_{i+1}) - A(t_i)] + \frac{\beta}{4\pi}\sum_{i=0}^{N}(\tau - t_i)^{-1/2}[B(t_{i+1}) - B(t_i)] \quad (3)$$

where N is the number of points used to subdivide the interval $[0,\tau]$ and t_i is the value of t at the end of the ith interval. This equation is simple to program for automatic computation and does not require the calculation of derivatives.

Comparison with Exact Solution

The method of Eq. (3) has been used to calculate the maximum sonic boom overpressure parameter at zero lift defined by Eq. (1) for the two configurations shown in Fig. 1. These configurations were chosen because it was possible to obtain an exact equation for the cross-sectional area distribution, hence making it possible to compute the exact value of the sonic boom overpressure parameter by use of Eqs. (1) and (2). A comparison of the results of Eq. (3) with the exact solution is shown in Table 1. The singularity at the upper limit of integration was avoided by defining τ as $t_N + c(t_N - t_{N-1})$ for both configurations. A study of the effect of N and c on the sonic boom overpressure parameter for four different configurations (including those shown in Fig. 1) has indicated that $n = 600$ and $c = 0.3$ gave excellent agreement with the exact solution. The fact that this study has been done only for the zero-lift case does not restrict the usefulness of the method since the effect of lift would be included by representing the lift distribution by a cross-sectional area distribution added to the vehicle in exactly the same way that a wing would be added to a body as a circular cross-sectional area distribution.

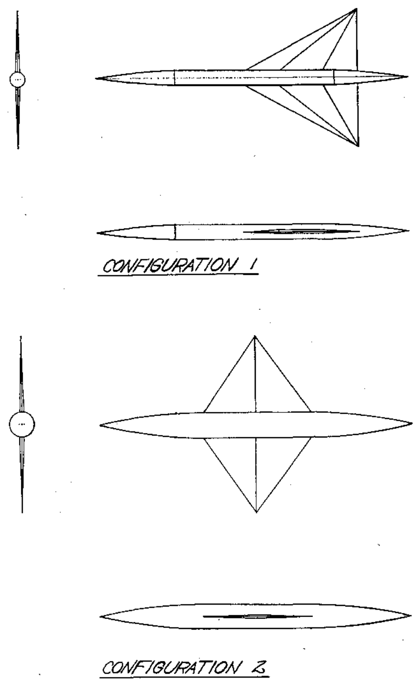

Fig. 1 Models used for the calculation of the sonic boom overpressure parameters.

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